TAX DEPRECIATION POLICY AND INVESTMENT THEORY

BY
VERNON L. SMITH

TECHNICAL REPORT NO. 109
FEBRUARY 27, 1962

PREPARED UNDER CONTRACT Nonr-225(50)
(NR-047-004)
FOR
OFFICE OF NAVAL RESEARCH

INSTITUTE FOR MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES
Applied Mathematics and Statistics Laboratories
STANFORD UNIVERSITY
Stanford, California

TAX DEPRECIATION POLICY AND INVESTMENT THEORY

by

Vernon L. Smith

TECHNICAL REPORT NO. 109

February 27, 1962

PREPARED UNDER CONTRACT Nonr-225(50)

(NR-047-004)

FOR

OFFICE OF NAVAL RESEARCH

INSTITUTE FOR MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES
Applied Mathematics and Statistics Laboratories
STANFORD UNIVERSITY
Stanford, California

TAX DEPRECIATION POLICY AND INVESTMENT THEORY*

by

Vernon L. Smith
Purdue and Stanford Universities

In a recent paper[1] I expressed the somewhat unconventional view that perhaps businesses should be permitted to depreciate or write off investment expenditures as rapidly as they please, including the extreme policy of treating such capital outlays as an ordinary business expense in the year incurred. The latter policy would be ostensibly[2] the most advantageous, given this freedom of choice. The only qualification mentioned was that provision should be made for the loss carry-over or carry-back of tax credits so that an otherwise rational managerial decision to incur heavy investment outlays in a year of low sales would not be artificially prejudiced by tax considerations. This is an issue of great currency as evidenced by the announced intention of the U. S. Treasury to liberalize present tax depreciation rules. The view that we might seriously consider permitting such wholesale tax depreciation freedom appears to be suggested by the present value theory of investment decisions. The issue might be approached with entirely different considerations and motivations in mind, but the present paper will be confined to a discussion of tax depreciation policy within the framework of the theory of investment of the firm. It is assumed throughout that it is desirable to impose taxes on business

* This work was supported in part by Office of Naval Research Contract Nonr-225(50) at Stanford University.

income in such a way that the structure of optimal investment decision rules is not altered by the tax.

By way of motivational argument it should be emphasized that the present value theory of investment decisions makes no use of the concept of depreciation as a write-off phenomenon, though it is possible to interpret various approaches to the so-called depreciation problem in terms of this theory. Taxless investment theory treats all receipts and outlays as cash inflows and outflows at the instant received or expended, and seeks to maximize the present worth of this lumpy net cash inflow. Sunk investments enter this stream only to the extent that they contribute to current receipts, current expenses, and provide lump-sum disinvestment receipts through salvage or resale. The capital outlays for sunk investments do not enter the future income stream and do not affect investment decisions. However, as we shall see, once taxes are introduced, investment decisions may be influenced by the cost of capital outlays for sunk investments. This is because the tax laws do not permit the cost of capital goods to be treated as an ordinary expense. The common practice is to permit capital costs to be written off or depreciated over time in accordance with some specified set of tax depreciation rules. It will be shown that this practice leads to bias in the form of investment decision rules different from those prevailing in the absence of a tax, that the bias is likely in the direction of delaying optimal investment timing, and that such biases can be removed by expensing investment outlays in the computation of taxable income. In deriving these results, we shall work first with an essentially static Preinreich-Lutz-Terborgh replacement model, then with a more general dynamic model in which price, current input, investment level and investment timing are joint decision variables in maximizing the present value of net income after taxes.

1. Taxes in a Simple Replacement Model

Let $R_0(t)$ be the net revenue at time t of a sunk investment in a productive facility, and let $M_0(L_0)$ be its market value as a function of the additional time it is held. After L_0 years, the firm invests in a new asset, and thereafter every L years in a chain of assets, where $R(L_0 + kL,t)$ is the net revenue of the $k+1^{th}$ member of this chain. The shift parameter k permits the effect of technological change to be represented. The initial investment in each of these future assets is C and they are assumed to bring a market price $M(L)$ after being held L years. The firm is assumed to maximize[3]

$$(1) \qquad V = \int_0^{L_0} R_0(t)e^{-rt}dt + M_0(L_0)e^{-rL_0} + e^{-rL_0}V' \quad ,$$

where

$$(2) \qquad V' = \sum_{k=0}^{\infty} e^{-rkL}\left\{\int_0^L R(L_0 + kL,t)e^{-rt}dt - C + M(L)e^{-rL}\right\}$$

with respect to L_0 and L. By setting $\frac{\partial V}{\partial L_0} \leq 0$, and $\frac{\partial V}{\partial L} = 0$, the optimal replacement condition can be written

$$(3) \qquad \frac{1}{r}[R_0(L_0) + M_0'(L_0) + \frac{\partial V'}{\partial L_0}] - V' \leq M_0(L_0) \quad ,$$

where V' now stands for its optimal value, that is, (2) is evaluated at the L for which $\frac{\partial V}{\partial L} = 0$. If the inequality holds in equilibrium, then L_0 is zero, that is, the incumbent asset is overdue for replacement. Statement (3) is the "programming" form of the familiar conditions for replacing an asset. An old asset should be replaced by its most attractive alternative, when the net contribution to the present worth of the firm caused by holding the asset an additional year does not exceed the market value of the asset.

In considering the effect of taxes, suppose we assume that investment outlays are expensed in arriving at taxable income, and that a constant tax rate α is applied to such income. Then at time t, where $0 \leq t < L_0$, the tax paid is $\alpha R_0(t)$; at L_0, the tax is $\alpha R(L_0,0) + \alpha M_0(L_0) - \alpha C$; at $L_0 + t$, $0 < t < L_0$, the tax is $\alpha R(L_0,t)$; at $L_0 + L$, the tax is $\alpha R(L_0 + L,0) + \alpha M(L) - \alpha C$, and so forth. By computing the present worth of these future tax payments and subtracting from V, given by (1), we get Π, the present worth of earnings net of taxes, which reduces to

$$\Pi = (1 - \alpha)V \quad . \tag{4}$$

Hence, maximizing Π with respect to L_0 and L leads to a decision rule identical with (3) obtained by maximizing the taxless present value V .

Consider next the policy of requiring the firm to follow any arbitrary investment depreciation rule for tax purposes. In general, such a rule specifies the write-offs as a function of time, and investment cost of the asset. Some specification is also normally made as to the appropriate bookkeeping adjustments when the asset is sold. Also, there is typically some minimum time period based, e.g., on average or "normal" asset life expectancy, during which the asset is to be depreciated.[4] Such rules define a tax depreciation function which, in general, we will write in the form $d_0 = d[C_0, M_0(L_0), u + t]$ for a sunk investment u years old at the opening of the planning period, and $d = d[C, M(L), t]$ for the chain of future replacement investments. Then the expression for the present value of profits after taxes becomes

$$\Pi^* = V - \alpha Y \quad , \tag{5}$$

where V is defined by (1), and the present value of taxable income Y is given by

$$Y = \int_0^{L_0} [R_0(t) - d_0] e^{-rt} dt + e^{-rL_0} Y' \quad , \tag{6}$$

where

$$Y' = \sum_{k=0}^{\infty} e^{-rkL} \int_0^{L} [R(L_0 + kL, t) - d] e^{-rt} dt \quad . \tag{7}$$

It is useful to define

$$A = \int_0^{L_0} d_0 \, e^{-rt} dt + M_0(L_0) e^{-rL_0} + e^{-rL_0} A' \quad , \tag{8}$$

where

$$A' = \sum_{k=0}^{\infty} e^{-rkL} \left\{ \int_0^{L} d e^{-rt} dt - C + M(L) e^{-rL} \right\} \quad . \tag{9}$$

Using (6)-(9), we can rewrite (5) in the form

$$\Pi^* = (1-\alpha) V + \alpha A \quad . \tag{10}$$

Now, maximizing Π^* with respect to L_0 and L gives

$$(1-\alpha) \frac{\partial V}{\partial L_0} \leq -\alpha \left[d_0 e^{-rL_0} + \int_0^{L_0} \frac{\partial d_0}{\partial L_0} e^{-rt} dt + M_0'(L_0) e^{-rL_0} - rM_0(L_0) e^{-rL_0} - re^{-rL_0} A' \right] \quad , \tag{11}$$

$$(12) \qquad (1-\alpha)\frac{\partial V}{\partial L} = \frac{-\alpha e^{-rL_0}}{1-e^{-rL}}\left[de^{-rL} + \int_0^L \frac{\partial d}{\partial L} e^{-rt}dt + M'(L)e^{-rL} - rM(L)e^{-rL} - re^{-rL}A'\right] .$$

We see immediately that (11) is not the same as the taxless decision rule (3) obtained from $\frac{\partial V}{\partial L_0} \leq 0$, and (12) will not in general give the taxless solution for L obtained from $\frac{\partial V}{\partial L} = 0$. In particular, note that the cost of the sunk investment, C_0, may enter the decision via the depreciation function d_0 .

To illustrate specifically what might be the effect of the tax depreciation component, shown on the right side of (11), on the investment decision, let us assume that the Treasury requires the use of straight line depreciation, and specifies that the proceeds from the sale of used equipment are to be counted (less any undepreciated portion of the original cost) as ordinary income at the time received. Then our tax depreciation functions can be written:

$$d_0 = d[C_0, M_0(L_0), u+t] = \begin{cases} \frac{C_0}{L_0^*} & , 0 \leq t < L_0^* - u \\ 0 & , L_0^* - u \leq t < L_0 \\ -M_0(L_0), & t = L_0 \end{cases}$$

$$d = d[C, M(L), t] = \begin{cases} \frac{C}{L^*} & , 0 \leq t < L^* \\ 0 & , L^* \leq t < L \\ -M(L) & , t = L \end{cases} ,$$

where L_0^* and L^* are the minimum write-off periods specified by the taxing authorities for the two types of facilities, and it is assumed that $L_0^* < u + L_0$, $L^* < L$. Then

$$A = \frac{C_0}{L_0^*}\left[\frac{1-e^{-r(L_0^*-u)}}{r}\right] + e^{-rL_0} A'$$

and

$$A' = \frac{C}{1-e^{-rL}}\left\{\frac{1-e^{-rL^*}}{rL^*} - 1\right\} < 0 \quad ,$$

where it is understood that when A and A' in (8) and (9) are evaluated at $t = L_0$ and $t = L$, the point values $-M_0(L_0)e^{-rL_0}$ and $-M(L)e^{-rL}$ are contributed by the corresponding integrals of d_0 and d .

Assuming an interior solution, (11) becomes

$$(1-\alpha)\frac{\partial V}{\partial L_0} = -\alpha\frac{\partial A}{\partial L_0} = \alpha re^{-rL_0} A' \quad . \tag{13}$$

Similarly, (12) becomes $(1-\alpha)\frac{\partial V}{\partial L} = -\alpha\frac{\partial A}{\partial L} = \frac{\alpha re^{-r(L+L_0)}A'}{1-e^{-rL}}$.[5] By differentiating these conditions in the usual way we can determine expressions for $\frac{\partial L_0}{\partial \alpha}$ and $\frac{\partial L}{\partial \alpha}$, but, in general, the signs of these derivatives are not unambiguously positive or negative. However, if we assume that the present equipment is like the replacement equipment, then we have only the one decision variable, $L = u + L_0$. Maximizing V' gives $(1-\alpha)\frac{\partial V'}{\partial L} = -\alpha\frac{\partial A'}{\partial L}$, and differentiating, we get

$$\frac{\partial L}{\partial \alpha} = -\frac{\frac{1}{1-\alpha}\frac{\partial A'}{\partial L}}{(1-\alpha)\frac{\partial^2 V'}{\partial L^2} + \alpha\frac{\partial^2 A'}{\partial L^2}} > 0,$$ since the denominator must be negative

for a maximum and $\frac{\partial A'}{\partial L} = - \frac{\alpha re^{-rL}A'}{1-e^{-rL}} > 0$. But note that if we have no taxes $(\alpha = 0)$ or, alternatively, if we expense capital outlays, we have $\frac{\partial V'}{\partial L} = 0$ in equilibrium. Hence, the partial equilibrium effect of straight line tax depreciation over a period shorter than the optimal life of equipment is to postpone reinvestment, as against optimal reinvestment under no taxes or taxes levied on income net of expensed capital outlays.

Using equations (11) and (12), it is reasonable to inquire as to whether it is possible to find tax depreciation functions which produce no bias in the investment decision rules. The conditions for such bias not to appear are obtained by setting $\frac{\partial A}{\partial L_0}$, the right side of (11), equal to zero, and $\frac{\partial A}{\partial L}$, the right side of (12), equal to zero. Solutions to the resulting differential equation conditions on d_0 and d can be written:

$$(14) \qquad \int_0^L d[C,M(L),t]e^{-rt}dt = C - M(L)e^{-rL} \quad ,$$

$$(15) \qquad \int_0^{L_0} d[C_0,M_0(L_0),u + t]e^{-rt}dt = C_0e^{ru} - M_0(L_0)e^{-rL_0} - \int_0^u d[C_0,M_0(L_0),t]e^{-rt}dt \quad ,$$

in which we make use of the administrative constraint that the depreciation rules are not to be different for sunk and replacement investments.[6] The solutions (14) and (15) can be verified by differentiating and substituting into the right sides of (11) and (12) to yield zero. Hence, any depreciation function whose present value over the optimal life of the asset is equal to the asset's cost plus the present worth of its salvage or resale value at the end of its life, has the property that it will not alter the investment decision rules. This result, though not perhaps very unexpected, also does not seem very useful in providing neutral tax depreciation guidelines within the framework of present tax depreciation policy. The requirement that the write-off must occur over the optimal equipment life for each firm, rather than some industry average, is hardly practical. For example, one tax depreciation function satisfying (14) and (15) is

$$d = \frac{rC-rM(L)e^{-rL}}{1-e^{-rL}}, \quad 0 \leq t < L, \quad \text{or} \quad d_0 = \frac{rC_0-rM_0(L_0)e^{-r(L_0+u)}}{1-e^{-r(L_0+u)}},$$

$0 \leq t \leq L_0$, which is just the annuity value of the investment cost net of salvage value. But to specify such a write-off allowance in the form of legal rules is hardly feasible, since L will normally vary among industries and firms for the same type of equipment. The simplest tax depreciation function satisfying (14) and (15) is of the form

$$d = \begin{cases} C & t = 0 \\ 0 & 0 < t < L \\ -M(L) & t = L \end{cases}$$

which is precisely the proposal for expensing capital outlays. This policy is quite easy to specify and to administer. We simply rule that, for tax purposes, the cost of an asset is deducted when that cost is incurred.

2. Taxes in a Dynamic Model

The previous model does not allow for level of investment decisions, and it does not distinguish explicitly technological data from economic (price) data in investment decisions. To show under more general conditions that investment decision neutrality requires assets to be expensed for tax purposes, we will first construct a dynamic model in which current price, current input, investment level, and investment timing policies are simultaneously determined.

Of the many possibilities that might be considered we will use a simple finite horizon model, in which it is assumed that at the opening of the planning period the firm has a sunk investment in $\bar{X}_1$ physical units of the capital facility, and that at most, one additional investment in X_2 units of the capital facility is to be considered in the planning period.[7] If the horizon is T, then the new investment is to occur at some time T_1, where $0 \leq T_1 \leq T$, to be determined. We also allow for the discard or sale of the sunk investment at T_1', $T_1 \leq T_1' \leq T$, to be determined. These specifications divide the planning interval into three operating periods. In the first period, $0 \leq t < T_1$, we have the short-run ex post production function $y(t) = f^1[x_1(t),\bar{X}_1]$, where $x_1(t)$ is a current input, and $\bar{X}_1$ is fixed. In the second period, $T_1 \leq t < T_1'$, the technological alternatives are described by the ex ante production function $y(t) = f^1[x_1'(t),\bar{X}_1] + f^2[x_2'(t),X_2]$, reflecting the parallel operation of old and new facilities. The functions f^1 and f^2 are assumed, in general, to differ and to show increasing returns. In the final operating period, $T_1' \leq t \leq T$, the old facility has been "phased out," and the production constraint becomes $y(t) = f^2[x_2(t),X_2]$. If W_1 and W_2 are the prices of the sunk and replacement investments, respectively, S_1 and S_2 are their fixed resale values, w is the price of the current input, and

$R[y(t),t]$ is the dynamic revenue function for the product, then the present value of the firm's profit can be written

$$(16)\quad V = \int_0^{T_1}\left\{R[f^1[x_1(t),\bar{X}_1],t] - w\, x_1(t)\right\} e^{-rt}dt$$

$$+ \int_{T_1}^{T_1'}\left\{ R[f^1[x_1'(t),\bar{X}_1] + f^2[x_2'(t),X_2],t] - w[x_1'(t) + x_2'(t)]\right\}e^{-rt}dt$$

$$+ \int_{T_1'}^{T}\left\{R[f^2[x_2(t),X_2],t] - w\, x_2(t)\right\} e^{-rt}dt$$

$$- W_2X_2\, e^{-rT_1} + S_1\bar{X}_1\, e^{-rT_1'} + S_2X_2\, e^{-rT} .$$

Price, $p(t) = \frac{R[y(t),t]}{y(t)}$, and current input rates in each period are assumed to be instantaneously variable, and their time paths are to be chosen, while the planning period levels of X_2, T_1, and T_1' are to be chosen. From the Euler conditions, we can write the following dynamic marginal revenue productivity conditions on the time path variables:

$$(17)\qquad \frac{\partial R}{\partial y} = \frac{w}{f_1^1[x_1(t),\bar{X}_1]} \qquad ; \quad 0 \leq t < T_1$$

$$(18)\qquad \frac{\partial R}{\partial y} = \frac{w}{f_1^1[x_1'(t),\bar{X}_1]} = \frac{w}{f_1^2[x_2'(t),X_2]} \qquad ; \quad T_1 \leq t < T_1'$$

$$(19)\qquad \frac{\partial R}{\partial y} = \frac{w}{f_1^2[x_2(t),X_2]} \qquad ; \quad T_1' \leq t \leq T$$

From setting the derivatives of V with respect to T_1, T_1' and X_2 equal to zero, we write the following necessary conditions for maximal investment level and for the timing of new investment and the sale of the old capital facility:

$$(20)\quad \int_{T_1}^{T_1'} \frac{wf_2^2[x_2'(t),X_2]e^{-rt}dt}{f_1^2[x_2'(t),X_2]} + \int_{T_1'}^{T} \frac{wf_2^2[x_2(t),X_2]e^{-rt}dt}{f_1^2[x_2(t),X_2]} = W_2e^{-rT_1} - S_2e^{-rT}$$

$$(21)\quad \frac{1}{r}\left\{R_2'(T_1) - R_1(T_1) + w[x_1(T_1) - x_1'(T_1) - x_2'(T_1)]\right\} = W_2X_2$$

$$(22)\quad \frac{1}{r}\left\{R_2'(T_1') - R_2(T_1') + w[x_2(T_1') - x_1'(T_1') - x_2'(T_1')]\right\} = S_1\bar{X}_1 \quad ,$$

where R_1, R_2', and R_2 denote respectively the revenue functions in the three integrals of (16). In the notation f_1^1, f_2^2, etc., the subscripts refer to the derivative with respect to the first or second argument of the production function, e.g., $f_1^2[x_2'(t),X_2] = \frac{\partial f^2}{\partial x_2'}$.

Equations (17)-(19) are the familiar conditions for equating instantaneous marginal cost and marginal revenue. The second equality in (18) also expresses the equi-marginal cost loading condition for multiple parallel facilities. (20) expresses the less familiar condition that the size of an additional capital facility is expanded until the present worth of the operating cost savings effected by an increment of the capital equals the present worth of the net cost of that increment (capital cost net of discounted resale value). (21) says that if a new facility is to be purchased, it must be at that point in time, T_1, when the capitalized value of the initial gain in net operating revenue equals the investment outlay for the facility. The discard condition (22) is symmetrical to (21). It requires an old asset to be discarded when the capitalized value of the initial loss in net operating revenue equals the resale value of the asset. These conditions could be Kuhn-Tuckerized by adding inequalities, but this refinement is not necessary to the purpose at hand. For example, if we had "<" instead of "=" in (21), then $T_1 = 0$, and the new facility would be added at the opening of the planning interval. It should be remarked that there may be many solutions satisfying these necessary conditions. In particular, if the requirements function $y(t)$ is not monotone, one should anticipate the likelihood that several T_1 and T_1' values might satisfy (21) and (22). Such difficulties seem inevitable in dynamic decision problems.

Returning now to the tax depreciation problem, it is clear that if all capital outlays and receipts are expensed for income tax calculations, then profit after taxes is $\Pi = (1-\alpha)V$, and maximizing Π gives the conditions (16)-(21). On the other hand, suppose the Treasury specifies that an asset must be depreciated over some write-off interval. Then we might express the general tax depreciation functions for the two types of assets in the form $d_1 = d(W_1\bar{X}_1, S_1\bar{X}_1, u+t)$ and $d_2 = d(W_2X_2, S_2X_2, t)$. Profit after taxes is now $\Pi^* = (1-\alpha)V + \alpha A$, with V given by (16) and A defined by

$$(23)\quad A = \int_0^{T_1'} d_1 e^{-rt} dt + \int_{T_1}^{T} d_2 e^{-rt} dt - W_2X_2 e^{-rT_1} + S_2\bar{X}_2 e^{-rT_1'} + S_2X_2 e^{-rT} .$$

Maximizing Π^* with respect to the time path variables gives the same decision rules (17)-(19) obtained by maximizing V. But with respect to X_2, T_1, and T_1', we now get

$$(24)\quad \left(\frac{1-\alpha}{\alpha}\right) \frac{\partial V}{\partial X_2} = W_2 e^{-rT_1} - S_2 e^{-rT} - \int_{T_1}^{T} \frac{\partial d_2}{\partial X_2} e^{-rt} dt \quad ,$$

$$(25) \qquad \left(\frac{1-\alpha}{\alpha}\right)\frac{\partial V}{\partial T_1} = [d(W_2X_2, S_2X_2, T_1) - r\, W_2X_2]e^{-rT_1}$$

$$(26) \qquad \left(\frac{1-\alpha}{\alpha}\right)\frac{\partial V}{\partial T_1'} = [d(W_1\bar{X}_1, S_1\bar{X}_1, T_1' + u) + rS_1\bar{X}_1]e^{-rT_1'} \quad ,$$

which again introduces tax depreciation components into the investment decision rules.

Conditions on the depreciation functions that eliminate this tax bias are obtained by setting the right side of equations (24)-(26) each equal to zero. A solution to the resulting differential equations can be written:

$$(27) \qquad \int_0^{T_1'} d(W_1\bar{X}_1, S_1\bar{X}_1, u+t)e^{-rt}dt = -\, S_1\bar{X}_1\; e^{-rT_1'} + F_1(W_1\bar{X}_1, S_1\bar{X}_1, u)$$

$$(28) \qquad \int_{T_1}^{T} d(W_2X_2, S_2X_2, t)e^{-rt}dt = W_2X_2\; e^{-rT_1} - S_2X_2\; e^{-rT} \quad ,$$

which can be verified by differentiating and substituting into the conditions obtained by setting the right side of (24)-(26) equal to zero. Making use of the fact that the taxing authorities cannot specify different depreciation allowances on sunk and future investments, the arbitrary function F_1 becomes $F_1 = W_1\bar{X}_1e^{ru} - \int_0^u d(W_1\bar{X}_1, S_1\bar{X}_1, t)e^{-rt}dt$, since $\int_0^{u+T_1'} de^{-rt}dt = W_1\bar{X}_1 - S_1\bar{X}_1e^{-r(T_1'+u)}$ is the counterpart of (28) for the sunk investment. The annuity depreciation formulas, $d_1 = \dfrac{rW_1\bar{X}_1 - rS_1\bar{X}_1e^{-r(T_1'+u)}}{1-e^{-r(T_1'+u)}}$, or $d_2 = \dfrac{rW_2X_2 - rS_2X_2e^{-r(T-T_1)}}{1-e^{-r(T-T_1)}}$ satisfy (27) and (28), and are natural candidates, but again would require the authorities to specify the optimal lives $T_1' + u$ and $T - T_1$ for each industry and firm. Under the simpler expensing approach we would have

$$d_2 = \begin{cases} W_2X_2 & t = 0 \\ 0 & 0 < t < T-T_1 \\ -S_2X_2 & t = T-T_1 \end{cases}$$

and similarly for d_1, which takes into account in a decentralized way the fact that optimal equipment life may vary from one firm to another.

3. Concluding Comments

The phenomena of business income taxes would appear to be a fact of life that is here to stay. We have attempted to show that if such taxes are levied on a concept of net business income that requires the specification of rules governing the manner in which capital outlays are to be charged as a current expense over time, such rules will, in general, introduce an artificial influence on investment decision formulas. Furthermore, there appears to exist no administratively feasible way to specify neutral write-off rules except to define taxable income as gross income minus all cash outlays including investment. This amounts to permitting businesses to fully expense capital expenditures for tax purposes, and represents the maximum rate of accelerated depreciation. This procedure recognizes that ultimately profits are the difference between total cash receipts and total cash outlays, however one might arbitrarily allocate short-run net cash receipts between something to which the name "profit" is given and something which is labeled "depreciation." Eventually, all depreciation schemes wash down to the same long-run net cash profit, and it is from this net profit that taxes must be paid.[8] Somehow, this very simple idea gets lost in the immensely complicated institutional mysteries of depreciation accounting.

Perhaps the most valuable advantage of fully expensing capital outlays is that of introducing administrative and clerical simplicity where there has tended to exist great complication. Trade sources frequently report that businesses keep at least two sets of books, one of which is designed specifically to solve the decision problems created by tax depreciation accounting. One finds it difficult to see what might be the social benefits of such activity.

One final point deserves to be made. In current discussions some have argued that faster tax depreciation write-offs should be allowed to give the growing firm an advantage. Our analysis suggests that the write-offs should be fully accelerated, not to give anyone an advantage but to eliminate an existing disadvantage in the sense that investment decision rules are distorted. Also bear in mind that many kinds of investment—probably our most important kinds—have always been expensed. I refer to investment by businesses in the training and further education of technical and scientific employes, investment in product research and development, and advertising outlays, all of which are expenditures designed to increase future earnings. Present tax write-off policies can hardly be said to have the same impact on a railroad or metal-working firm as on a pharmaceutical or electronics firm whose investment in knowledge is relatively far greater and more crucial than their outlays for durable goods.

FOOTNOTES

1. Smith, V. L. "Depreciation and Investment Theory," ONR Technical Report No. 105, Institute for Mathematical Studies in the Social Sciences, December, 1961.

2. It would seem obvious that the profit motive should drive corporate management to depreciate assets as rapidly as is permitted under law. However, many have expressed the opinion that management has been slow to adopt the liberalized depreciation privileges provided in the Internal Revenue Code of 1954 [see, e.g., the proceedings of the symposium <u>Depreciation and Taxes</u> (Princeton: Tax Institute, 1959), pp. 130, 172]. J. Barlow (<u>Ibid</u>., pp. 131-140) mentions several reasons for this. Besides ignorance or misunderstanding of the after-tax benefits of accelerated depreciation, there is the problem that some managements seem to view depreciation deductions as a cost which adversely affects profits, and the opinions of bankers and shareholders. This last reason could have substance if the availability of funds is influenced by short-term earnings which, under accelerated depreciation, tend to be depressed in the early years of an investment, but increased in later years. Also there are the vulgar facts that existing stock options to management may be less valuable, and profit-sharing programs for management will be less attractive to present management [cf. T. N. McDade, <u>Ibid</u>., Chapter III, p. 33 and passim].

3. Smith, <u>op</u>. <u>cit</u>., p. 9.

4. Such "normal" life guidelines are provided in the U.S. Treasury's Bulletin F.

5. The equilibrium conditions are different if the minimum write-off periods exceed the optimal life of equipment. If $L_o < L_o^* + u$, $L < L^*$, and we assume that the "undepreciated" portion of the asset is deducted at the time of replacement, then

$$d_o = \begin{cases} \frac{C_o}{L_o^*}, & 0 \leq t < L_o \\ - M_o(L_o) + \frac{C_o}{L_o^*}(L_o^* - u - L_o), & t = L_o \end{cases}$$

and similarly for the replacement investment. If one computes A and A' for this case the condition (13) becomes

$(1 - \alpha)\frac{\partial V}{\partial L_o} = \frac{\alpha r C_o}{L_o^*}(L_o^* - u - L_o) + \alpha r e^{-rL_o} A'$. Note that the cost of the sunk investment influences its replacement, which was not the case in (13). This demonstrates how sensitive are the decision rules to the parameters of a given write-off policy, as well as the policies.

6. Obviously the Treasury cannot specify a different write-off policy for sunk than for future replacement assets to be purchased. (14) has the same form as (15) since

$$\int_o^{L_o} d[C_o, M_o(L_o), u + t]e^{-r(u+t)}\, dt$$

$$+ \int_o^{u} d[C_o, M_o(L_o), t]e^{-rt}\, dt = \int_o^{u+L_o} d[C_o, M_o(L_o), t]e^{-rt}\, dt \ .$$

7. This is a profit maximizing extension of the models discussed in Smith, V. L., Investment and Production, (Harvard University Press: Cambridge, 1961), Chapter XI, especially pp. 293-298.

8. Of course there is the problem that tax payments are "delayed." But the term "delay" is used only because it is generally believed that the present non-expensed approach to tax depreciation is "natural." One could equally argue, that under existing practices, corporations are making interest-free advance payments to the Treasury; that the real financial burden or "cost" (in the sense of foregoing "consumption" distribution to stockholders) of an investment occurs in the year of the capital outlay, and therefore expensing is "natural." The argument, of course, depends ultimately upon how one proposes to measure income.

But, however one interprets this delay, the fact remains that Treasury revenues are adversely affected. My proposal for making up this loss has been to remove the differential treatment of capital gains. In this respect it is worth noting that a recent survey of 150 executives in 51 major corporations has revealed that "managements are willing to give up capital gains treatment of gains arising from the sale of depreciable plant and equipment in order to lessen the impact on Treasury revenue of needed tax depreciation reform." See R. Milroy, D. Istvan and R. Powell, "The Tax Depreciation Muddle," <u>The Accounting Review</u>, Vol. 36, No. 4, October, 1961, p. 540.

STANFORD UNIVERSITY

TECHNICAL REPORT DISTRIBUTION LIST

CONTRACT Nonr-225(50)

(NR 047-004)

Office of Naval Research
Branch Office
346 Broadway
New York 13, N. Y. 1

Office of Naval Research
Branch Office
1030 E. Green Street
Pasadena 1, California 1

Office of Naval Research
Branch Office
1000 Geary Street
San Francisco 9, California 1

Office of Naval Research
Navy No. 100
Fleet Post Office
New York, New York 2

Office of Naval Research
Logistics Branch, Code 436
Dept. of the Navy
T3-Bldg.
Washington 25, D. C. 10

Headquarters, USAF
Attn: AFASC - 6F
Washington 25, D. C. 1

Logistics Research Project
The George Washington University
707 - 22nd Street, N. W.
Washington 7, D. C. 1

Operations Research Office
The Johns Hopkins University
6935 Arlington Road
Bethesda 14, Maryland 1

United States Air Force
Air University Library
Maxwell Air Force Base, Alabama 1

U. S. Naval Supply Research and
Development Facility
Naval Supply Depot
Bayonne, New Jersey 1

Weapons Systems Evaluation Group
Pentagon Bldg.
Washington 25, D. C. 1

Ames Aeronautical Laboratory
Moffett Field, California
Attn: Technical Library 1

Armed Services Technical Information
Agency
Arlington Hall Station
Arlington 12, Virginia 10

The Director
Naval Research Laboratory
Washington 25, D. C.
Attn: Tech. Information Office 1

Chief, Bureau of Supplies
and Accounts
Advanced Logistics Research Division
(Code W3)
Department of the Navy
Washington 25, D. C. 1

Naval War College
Logistics Dept., Luce Hall
Newport, Rhode Island 1

Director
National Security Agency
Attn: C3141(Rm.2C087)
Fort George G. Meade, Maryland 1

Director
National Science Foundation
Washington 25, D. C. 1

NACA Office for Aeronautics
1724 F. Street, N. W.
Washington 25, D. C.
Attn: Chief, Office of
Aeronautical Engineering 1

Bureau of Supplies and Accounts Code OW
Department of the Navy
Washington 25, D. C. 1

Director
Operations Evaluation Group
Office of Chief
of Naval Operations (OP-03EG)
Navy Dept.
Washington 25, D. C. 2

Headquarters
Oklahoma City Air Materiel Area
United States Air Force
Tinker Air Force Base, Oklahoma 1

Industrial College of the
Armed Forces
Fort Lesley J. McNair
Washington 25, D. C.
Attn: Mr. L. L. Henkel 1

Institute for Defense Analyses
Communications Research Division
Von Neumann Hall
Princeton, New Jersey 1

Library
American Power Jet Company
705 Grand Avenue
Ridgefield, New Jersey 1

Commanding Officer
Office of Naval Research
Branch Office
86 E. Randolph Street
Chicago 1, Illinois 1

Superintendent
U. S. Naval Postgraduate School
Attn: Library
Monterey, California 1

Electronic Computer Division
Code 280
Bureau of Ships
Department of the Navy
Washington 25, D. C. 2

The RAND Corporation
1700 Main Street
Santa Monica, California 1

Management Controls, Ref. Library
Remington Rand,
Div. of Sperry Rand Corp.
315 Fourth Avenue
New York 10, N. Y. 1

University of Washington
Inst. for Economic Research
Seattle 5, Washington 1

University of California
Bureau of Business and
Economic Research
Berkeley 4, California 1

Mr. Adam Abruzzi
Applied Statistics Group
Department of Management
Stevens Institute of Technology
Hoboken, New Jersey 1

Dr. S. G. Allen
Stanford Research Institute
Menlo Park, California 1

Professor Leland Bach
Department of Economics
Carnegie Institute of Technology
Pittsburgh 13, Pennsylvania 1

Professor E. F. Beckenbach
Department of Mathematics
University of California
Los Angeles 24, California 1

Dr. Martin J. Beckmann
Department of Economics
Brown University
Providence 12, Rhode Island 1

Dr. Richard Bellman
The RAND Corporation
1700 Main Street
Santa Monica, California 1

Professor Max R. Bloom
College of Business Administration
Syracuse University
Syracuse 10, New York 1

Dean L. M. K. Boelter
School of Engineering
University of California
Los Angeles 24, California 1

Professor James N. Boles
University of California
Agricultural Experiment Station
Berkeley 4, California 1

Professor O. H. Brownlee
Department of Economics
University of Minnesota
Minneapolis 14, Minnesota 1

Professor S. S. Cairns
Head, Dept. of Mathematics
University of Illinois
Urbana, Illinois 1

Mr. Samuel L. Carrell
7022 Tokalon Drive
Dallas, Texas 1

Professor A. Charnes
The Technological Institute
Northwestern University
Evanston, Illinois 1

Professor John S. Chipman
Department of Economics
University of Minnesota
Minneapolis 14, Minnesota 1

Professor Carl Crist
Behavioral Sciences
202 Junipero Serra
Stanford, California 1

Professor Randolph Church
U. S. Naval Postgraduate School
Monterey, California 1

Professor C. W. Churchman
School of Business Administration
University of California
Berkeley 4, California 1

Professor W. W. Cooper
Director of Research
Department of Economics
Carnegie Institute of Technology
Pittsburgh 13, Pennsylvania 1

Professor Gerald Debreu
Cowles Foundation for Research in Eco.
Yale Station, Box 2125
New Haven, Connecticut 1

Professor Robert Dorfman
Department of Economics
Harvard University
Cambridge 38, Massachusetts 1

Mr. Louis Doros
Management Engineering Staff
Bureau of Yards and Docks
Navy Department
Washington 25, D. C. 1

Rear Adm. H. E. Eccles, USN, Ret.
101 Washington Street
Newport, Rhode Island 1

Mr. Daniel Ellsberg
The RAND Corporation
1700 Main Street
Santa Monica, California 1

Alain Enthoven
3009 Holly Street
Alexandria, Virginia 1

Professor Eberhard Fels
445 CL
University of Pittsburgh
Pittsburgh 13, Pennsylvania 1

Contract Nonr-225(50)
November 1961

Dr. Merrill M. Flood
Mental Health Research Institute
205 North Forest Avenue
Ann Arbor, Michigan 1

Professor David Gale
Department of Mathematics
Brown University
Providence, Rhode Island 1

Mr. Murray Geisler
The RAND Corporation
1700 Main Street
Santa Monica, California 1

Professor B. Gelbaum
Department of Mathematics
University of Minnesota
Minneapolis 14, Minnesota 1

Giannini Foundation Ground-Water Studies
Giannini Foundation of Agri. Economics
University of California
Berkeley 4, California 1

Mr. R. J. D. Gillies
Marketing Research Department
Olin Mathieson Chemical Corp.
275 Winchester Avenue
New Haven 4, Connecticut 1

Dr. R. E. Gomory
IBM Research
Yorktown Heights, New York 1

Professor Zvi Griliches
Department of Economics
University of Chicago
Chicago 37, Illinois 1

Dr. Isadore Heller
Navy Management Office
Washington 25, D. C. 1

Dr. Theodore E. Harris
The RAND Corporation
1700 Main Street
Santa Monica, California 1

Dr. John C. Harsanyi
Department of Economics
Wayne State University
Detroit, Michigan 1

Professor M. R. Hestenes
Department of Mathematics
University of California
Los Angeles 24, California 1

Professor C. Hildreth
Michigan State University
East Lansing, Michigan 1

Mr. C. J. Hitch
The RAND Corporation
1700 Main Street
Santa Monica, California 1

Dr. Alan J. Hoffman
IBM Research Center
Box 218
Yorktown Heights. New York 1

Professor Austin Hoggatt
School of Business Administration
University of California
Berkeley 4, California 1

Dr. C. C. Holt
Grad School of Industrial Administration
Carnegie Institute of Technology
Pittsburgh 13, Pennsylvania 1

Dr. John W. Hooper
Cowles Foundation for Research
in Economics
Yale Station, Box 2125
New Haven, Connecticut 1

Professor H. Hotelling
Dept. of Mathematical Statistics
University of North Carolina
Chapel Hill, North Carolina 1

Professor H. S. Houthakker
Department of Economics
Harvard University
Cambridge 38, Massachusetts 1

Dr. H. M. Hughes
Department of Biometrics
School of Aviation Medicine
U.S.A.F.
Randolph Field, Texas 1

Professor Leonid Hurwicz
Department of Economics
University of Minnesota
Minneapolis 14, Minnesota 1

Professor J. R. Jackson
Management Sciences Res. Proj.
University of California
Los Angeles 24, California 1

Professor Dale W. Jorgenson
Department of Economics
University of California
Berkeley 4, California 1

Johns Hopkins University Library
Acquisitions Department
Baltimore 18, Maryland 1

Capt. Walter H. Keen, USN
Office of Naval Research
Code 400
Department of Navy
Washington 25, D. C. 1

Professor Lawrence Klein
Department of Economics
University of Pennsylvania
Philadelphia, Pennsylvania 1

Professor T. C. Koopmans
Cowles Foundation for Research
in Economics
Yale Station, Box 2125
New Haven, Connecticut 1

Professor Harold W. Kuhn
Department of Mathematics
Bryn Mawr College
Bryn Mawr, Pennsylvania 1

Dr. Howard Laitin
The RAND Corporation
1700 Main Street
Santa Monica, California 1

Dr. R. F. Lanzellotti
Department of Economics
Washington State College
Pullman, Washington 1

Professor C. E. Lemke
Department of Mathematics
Rensselaer Polytechnical Institute
Troy, New York 1

Professor W. W. Leontief
Department of Economics
Harvard University
Cambridge 38, Massachusetts 1

Dr. Bernhard Lieberman
Center for International Affairs
6 Divinity Ave.
Harvard University
Cambridge 38, Massachusetts 1

Professor S. B. Littauer
Dept. of Industrial Engineering
Columbia University
New York 27, New York 1

Dr. R. Duncan Luce
Dept. of Psychology
University of Pennsylvania
Philadelphia, Pennsylvania 1

Dr. Craig A. Magwire
2954 Winchester Way
Rancho Cordova, California 1

Benoit Mandelbrot
International Business
Machines Corp.
Research Center
P. O. Box 218, Lamb Estate
Yorktown Heights, New York 1

Professor Julius Margolis
Department of Business Administration
University of California
Berkeley 4, California 1

Mr. Harry M. Markowitz
Logistics Department
The RAND Corporation
1700 Main Street
Santa Monica, California 1

Professor Jacob Marschak
School of Business Administration
University of California
Los Angeles 24, California 1

Thomas Marschak
School of Business Administration
University of California
Berkeley 4, California 1

Professor Lionel W. McKenzie
Department of Economics
University of Rochester
River Campus Station
Rochester 20, New York 1

Mr. Joseph Mehr
Head, Operations Research Desk
U. S. N. Training Device Center
Port Washington, L. I., New York 1

Dr. Richard A. Miller
4071 West 7th Street
Fort Worth 7, Texas 1

Professor Franco Modigliani
Grad. School of Industrial Administration
Carnegie Institute of Technology
Pittsburgh 13, Pennsylvania 1

Professor James Moore
Dept. of Industrial Engineering
Northeastern University
Boston 12, Mass. 1

Professor O. Morgenstern
Econometric Research Program
Princeton University
92-A Nassau Street
Princeton, New Jersey 1

D. E. Newham
Chief, Industrial Engineering Div.
Comptroller
Hdqrs, San Bernardino Air Materiel Area
Norton Air Force Base, California 1

Mr. M. L. Norden
Research Division
College of Engineering
New York University
New York 53, New York 1

Professor R. R. O'Neill
Department of Engineering
University of California
Los Angeles 24, California 1

Mr. Richard V. Palermo
Operations Research Department
Willow Run Laboratories
Ypsilanti, Michigan 1

Professor Stanley Reiter
Department of Economics
Purdue University
Lafayette, Indiana 1

Professor M. Riley
301 Stephens Memorial Hall
University of California
Berkeley 4, California 1

Professor D. Rosenblatt
Department of Statistics
The George Washington University
Washington 7, D. C. 1

Professor Murray Rosenblatt
Department of Mathematics
Brown University
Providence 12, Rhode Island 1

Professor A. E. Ross, Head
Department of Mathematics
University of Notre Dame
Notre Dame, Indiana 1

Professor Jerome Rothenberg
Department of Economics
Northwestern University
Evanston, Illinois 1

Professor Albert H. Rubenstein
Department of Industrial Engineering
Northwestern University
Evanston, Illinois 1

Professor Herman Rubin
Department of Statistics
Michigan State University
East Lansing, Michigan 1

Professor Richard S. Rudner
Department of Philosophy
Michigan State University
East Lansing, Michigan 1

Contract Nonr-225(50)
November 1961

Professor P. A. Samuelson
Department of Economics
M. I. T.
Cambridge 39, Massachusetts 1

Dr. I. Richard Savage
School of Business
Vincent Hall
University of Minnesota
Minneapolis 14, Minnesota 1

Professor Andrew Schutz, Jr.
Dept. of Industrial Engineering
Cornell University
Ithaca, New York 1

Professor H. A. Simon
Dept. of Industrial Administration
Carnegie Institute of Technology
Pittsburgh 13, Pennsylvania 1

Mr. J. R. Simpson
Bureau of Supplies and Accounts
Code SS
Arlington Annex
U. S. Department of Navy
Washington 25, D. C. 1

Professor Laurie Snell
Department of Mathematics
Dartmouth College
Hanover, New Hampshire 1

Professor Robert Solow
Department of Economics
Mass. Inst. of Technology
Cambridge 39, Massachusetts 1

Professor Robert Summers
Department of Economics
University of Pennsylvania
Philadelphia, Pennsylvania 1

Professor Gerald Thompson
Dept. of Industrial Administration
Carnegie Institute of Technology
Pittsburgh 13, Pennsylvania 1

Professor R. M. Thrall
Department of Mathematics
University of Michigan
Ann Arbor, Michigan 1

Professor L. M. Tichvinsky
Department of Engineering
University of California
Berkeley 4, California 1

Professor James Tobin
Council of Economic Advisers
Executive Office of The President
Washington 25, D. C. 1

Professor C. B. Tompkins
Dept. of Mathematics
University of California
Los Angeles 24, California 1

Professor A. W. Tucker
Fine Hall, Box 708
Princeton, New Jersey 1

Professor Hirofumi Uzawa
Behavioral Sciences
202 Junipero Serra
Stanford, California 1

Professor D. F. Votaw, Jr.
Department of Economics
Yale University
New Haven, Connecticut 1

Professor W. A. Wallis
207 Haskell Hall
University of Chicago
Chicago 37, Illinois 1

Professor J. L. Walsh
Department of Mathematics
Harvard University
Cambridge 38, Massachusetts 1

Dr. T. Whitin
School of Industrial Management
Mass. Institute of Technology
Cambridge 38, Massachusetts 1

Mr. Wayne A. Wickelgren
6393 - A Dana Street
Oakland 9, California 1

Mr. Philip Wolfe
The RAND Corporation
1700 Main Street
Santa Monica, California 1

Professor J. Wolfowitz
Department of Mathematics
Cornell University
Ithaca, New York 1

Professor Robert J. Wolfson
CEIR Inc.
9171 Wilshire Blvd.
Beverly Hills, California 1

Mr. Marshall K. Wood, Chief
Industrial Vulnerability Branch
Office of Ass't. Sec. of Defense
Washington 25, D. C. 1

Professor M. A. Woodbury
Department of Mathematics
New York University
New York 53, New York 1

<u>Distribution via ONR London</u>

Commanding Officer
Branch Office
Navy No. 100
Fleet Post Office
New York, New York

Professor Maurice Allais
62 Boulevard St. Michel (VI°)
Paris 6e, FRANCE 1

Mr. William M. Gorman
University of Birmingham
Birmingham, ENGLAND 1

Mr. Frank Hahn
Department of Economics
Churchill College
Cambridge, ENGLAND 1

Dr. Rudolf Hauser
Institut für angewandte Mathematik
Universität Bern
Bern, SWITZERLAND 1

M. Jacques Lesourne
Société des Mathematiques
Appliquées
33 rue Mogador
Paris 9e, FRANCE 1

Dr. Edmond Malinvaud
Institute National de la Statistique
et des Études Economiques
29, quai Branly
Paris, FRANCE 1

Professor Maurice McManus
Faculty of Commerce
and Social Science
University of Birmingham
Birmingham, England 1

Professor Dr. Heinz Sauermann
Institut für Fremdenverkehrswissenschaft
Grafstrasse 39
Frankfurt a.M., GERMANY 1

Professor Henri Theil
Econometric Institute
Netherlands School of Economics
Rotterdam, NETHERLANDS 1

<u>Distribution via ONR San Francisco</u>

U. S. Attaché
Attaché for Air (Tokyo, Japan)
APO - 500
San Francisco, California

Professor Michio Morishima
Institute of Social and Economic
Research
Osaka University
Toyonaka, Osaka
JAPAN 1

Takashi Negishi
112 Sanyacho Meguroku
Tokyo, JAPAN 1

Dr. Hukukane Nikaido
Institute of Social and Economic
Research
Osaka University
Toyonaka, Osaka
JAPAN 1

Dr. Tsunehiko Watanabe
Dept. of Economics
Gakuin University
Mejiro, Toshimaku
Tokyo, JAPAN 1

Professor Takuma Yasui
Department of Economics
Tohoku University
Sendai, JAPAN 1

<u>Other Foreign Addresses</u>

Jan Kmenta
School of Economics
University of N. S. Wales
Kensington, N. S. Wales, AUSTRALIA 1

Professor David Slater
Queen's University
Kingston, Ontario
CANADA 1

Professor D. V. T. Bear
Dept. Economics
University of Chicago
Chicago 37, Illinois 1

Additional copies for project leader
and assistants and reserve for future
requirements. 50

Contract Nonr-225(50)
November 1961
(21)